QVINE

DEVIANT LOGICS

Change of logic, change of subject

In the preceding chapter we discussed the bounds of logic. We considered where, within the totality of science that we accept, the reasonable boundary falls between what we may best call logic and what we may best call something else. We considered also, outside the firm area thus bounded, certain supplementary developments which we would include under the name of logic if we were to admit them into our total science at all. We did not consider any possible inroads on the firm area itself. This is our next topic: the possible abrogation of the orthodox logic of truth functions or of quantification in favor of some deviant logic.

The systems of orthodox logic are themselves many and varied. The differences among them are not such as make deviant logics. It is one logic variously expounded and variously serviced by computers or proof procedures. Demarcate the totality of logical truths, in whatever terms, and you have in those terms specified the logic. Which of these truths one chooses to designate as axioms, and what rules he devises for generating the rest of the logical truths from those axioms, is indifferent. Whether he elects the axiomatic style at all, or some other sort of proof procedure, or none, is again indifferent. The kind of deviation now to be considered, on the other hand, is of a more substantial kind. It is not just a change of methods of generating the class of logical truths, but a change of that class itself. It is not just a change of demarcation, either, between what to call logical truth and what to call extra-logical truth. It is a

question rather of outright rejection of part of our logic as not true at all.

It would seem that such an idea of deviation in logic is absurd on the face of it. If sheer logic is not conclusive, what is? What higher tribunal could abrogate the logic of truth functions or of quantification?

Suppose someone were to propound a heterodox logic in which all the laws which have up to now been taken to govern alternation were made to govern conjunction instead, and vice versa. Clearly we would regard his deviation merely as notational and phonetic. For obscure reasons, if any, he has taken to writing 'and' in place of 'or' and vice versa. We impute our orthodox logic to him, or impose it upon him, by translating his deviant dialect.

Could we be wrong in so doing? Could he really be meaning and thinking genuine conjunction in his use of 'and' after all, just as we do, and genuine alternation in his use of 'or', and merely disagreeing with us on points of logical doctrine respecting the laws of conjunction and alternation? Clearly this is nonsense. There is no residual essence of conjunction and alternation in addition to the sounds and notations and the laws in conformity with which a man uses those sounds and notations.

To turn to a popular extravaganza, what if someone were to reject the law of non-contradiction and so accept an occasional sentence and its negation both as true? An answer one hears is that this would vitiate all science. Any conjunction of the form '$p \,.\, {\sim}p$' logically implies every sentence whatever; therefore acceptance of one sentence and its negation as true would commit us to accepting every sentence as true, and thus forfeiting all distinction between true and false.

In answer to this answer, one hears that such a full-width trivialization could perhaps be staved off by making compensatory adjustments to block this indiscriminate deducibility of all sentences from an inconsistency. Perhaps, it is suggested, we can so rig our new logic that it will isolate its contradictions and contain them.

My view of this dialogue is that neither party knows what he is talking about. They think they are talking about negation, '$\sim$', 'not'; but surely the notation ceased to be recognizable as negation when they took to regarding some conjunctions of the form '$p \,.\, {\sim}p$' as true, and stopped regarding such sentences as implying all others. Here, evidently, is the deviant logician's predicament: when he tries to deny the doctrine he only changes the subject.

Logic in translation

Take the less fanciful case of trying to construe some unknown language on the strength of observable behavior. If a native is prepared to assent to some compound sentence but not to a constituent, this is a reason not to construe the construction as conjunction. If a native is prepared to assent to a constituent but not to the compound, this is a reason not to construe the construction as alternation. We impute our orthodox logic to him, or impose it on him, by translating his language to suit. We build the logic into our manual of translation. Nor is there cause here for apology. We have to base translation on some kind of evidence, and what better?

Being thus built into translation is not an exclusive trait of logic. If the natives are not prepared to assent to a certain sentence in the rain, then equally we have reason not to translate the sentence as 'It is raining'. Naturally the native's unreadiness to assent to a certain sentence gives us reason not to construe the sentence as saying something whose truth should be obvious to the native at the time. Data of this sort are all we have to go on when we try to decipher a language on the basis of verbal behavior in observable circumstances.

Still, logic is built into translation more fully than other systematic departments of science. It is in the incidence of obviousness that the difference lies. Preparatory to developing this point I must stress that I am using the word 'obvious' in an ordinary behavioral sense, with no epistemological overtones. When I call '1 + 1 = 2' obvious to a community I mean only that everyone, nearly enough, will unhesitatingly assent to it, for whatever reason; and when I call 'It is raining' obvious in particular circumstances I mean that everyone will assent to it in those circumstances.

It behooves us, in construing a strange language, to make the obvious sentences go over into English sentences that are true and, preferably, also obvious; this is the point we have been noting. Now this canon—'Save the obvious'—is sufficient to settle, in point of truth value anyway, our translations of *some* of the sentences in just about every little branch of knowledge or discourse; for some of them are pretty sure to qualify as obvious outright (like '1 + 1 = 2') or obvious in particular circumstances (like 'It is raining'). At the same time, just about every little branch of knowledge or discourse will contain other sentences which are not thus guaranteed true by translation, not being obvious.

But on this score logic is peculiar: every logical truth is obvious, actually or potentially. Each, that is to say, is either obvious as it stands or can be reached from obvious truths by a sequence of individually obvious steps. To say this is in effect just to repeat some remarks of Chapter 4: that the logic of quantification and identity admits of complete proof procedures, and some of these are procedures that generate sentences purely from visibly true sentences by steps that visibly preserve truth.

In a negative sense, consequently, logical truth is guaranteed under translation. The canon 'Save the obvious' bans any manual of translation that would represent the foreigners as contradicting our logic (apart perhaps from corrigible confusions in complex sentences). What is negative about this guarantee is that it does not assure that all our logically true sentences carry over into truths of the foreign language; some of them might resist translation altogether.

The law of excluded middle

One issue that calls for examination under the head of deviant logic has to do with the law of excluded middle, or *tertium non datur*. This law, which has been contested in some quarters, may be pictured variously:

(1) Every closed sentence is true or false,
(2) Every closed sentence or its negation is true,
(3) Every closed sentence is true or not true.

We may as well economize on components by explaining falsity as truth of the negation. This reduces (1) to (2). As for (3), it looks more modest than (2). What little it affirms continues to hold, unlike (2), even when we change 'closed sentence' to 'open sentence' or 'question' or 'command' and even when we change 'true' to 'brief' or 'musical'. The form of (3) is '$\forall x$ (if Fx then Gx or $\sim Gx$)', whose validity follows from that of 'p or $\sim p$'. Still, what does it mean to call 'p or $\sim p$' valid? Simply that it comes out true with any closed sentence in place of 'p'. But this amounts in effect to (2), it would seem, after all, so that the difference in strength between (2) and (3) is illusory. Schematically, the law of excluded middle is simply 'p or $\sim p$'.

These trivial latter lucubrations well illustrate the inanity of trying to discern equivalence in some sense within the domain of logical truth. Logical equivalence, as of Chapter 4, holds indiscriminately between all logical truths.

By the reasoning of a couple of pages back, whoever denies the law of excluded middle changes the subject. This is not to say that he is wrong in so doing. In repudiating 'p or $\sim p$' he is indeed giving up classical negation, or perhaps alternation, or both; and he may have his reasons.

One setting where classical negation and alternation fall away is many-valued logic. This kind of logic was developed somewhat by C. S. Peirce in the last century, and independently later by Łukasiewicz. It is like the logic of truth functions except that it recognizes three or more so-called truth values instead of truth and falsity. Primarily the motivation of these studies has been abstractly mathematical: the pursuit of analogy and generalization. Studied in this spirit, many-valued logic is logic only analogically speaking; it is uninterpreted theory, abstract algebra.

Sometimes, however, three-valued logic is envisaged as an improved logic. Its three values are called truth, falsity, and something intermediate. A construction called negation carries so-called truths into falsehoods, falsehoods into truths, and intermediates into intermediates. On these terms the law of excluded middle palpably fails. But we must remember, even while honoring this deviant logic as genuine logic, that the terminology 'true', 'false', and 'negation' carries over into it from our logic only by partial analogy. The failure of the law is, insofar, nominal.

By projecting the terminology along different analogies, might the law of excluded middle be nominally salvaged here still? It seems not. Call the new truth values 1, 2, 3. We can indeed group the values 2 and 3 under the joint heading 'false', and thus count each closed sentence still as "true" or "false." Or if, better, we continue to economize on terms by explaining falsity as truth of negation, the suggestion comes to this: value 1 is truth, and negation is to lead from the values 2 and 3 to 1 and from 1 to 2 or 3. But, if negation is to be a truth function at all, we must make up our mind: it must lead from 1 always to 2 or always to 3. Then, however, we forfeit the law of double negation. For, say negation leads from 1 always to 2; then double negation leads from 3 to 1 to 2 rather than back to 3. Thus we nominally salvage the law of excluded middle only by forfeiting double negation. Try what we will, three-valued logic turns out true to form: it is a rejection of the classical true-false dichotomy, or of classical negation.

It is hard to face up to the rejection of anything so basic. If anyone questions the meaningfulness of classical negation, we are tempted to say in defense that the negation of any given closed sentence is *explained* thus: it is true if and only if the given sentence is not true. This, we may feel, meets the charge of meaninglessness by providing meaning, and indeed a meaning that assures that any closed sentence or its negation is true. However, our defense here begs the question; let us give the dissident his due. In explaining the negation as true if and only if the given sentence is not true, we use the same classical 'not' that the dissident is rejecting.

Debate about the dichotomy

Let us grant, then, that the deviant can coherently challenge our classical true-false dichotomy. But why should he want to? Reasons over the years have ranged from bad to better. The worst one is that things are not just black and white; there are gradations. It is hard to believe that this would be seen as counting against classical negation; but irresponsible literature to this effect can be cited.

The next to worst one is a confusion between knowledge and truth. Certainly there is a vast intermediate domain of sentences between those that we know or even believe to be true and those that we know or believe to be false; but we can still hold that each of those intermediate sentences is either true, unbeknownst to us, or false unbeknownst to us. Perhaps part of the trouble is a confusion between (a) knowing something to be true or false and (b) knowing something to be true or knowing it to be false.

A more respectable reason for protesting the dichotomy has to do with the paradoxes of set theory and semantics. Thus take again Russell's paradoxical class $\{x: \sim(x \in x)\}$, and the sentence that says this class is a member of itself. The proposal is that we allow this and similar sentences the middle truth value. The equivalence, once so vexatious, of these sentences to their own negations, can thereupon be received with equanimity—negation now being, of course, the reformed negation of three-valued logic.

This proposal stems from Bočvar, 1939. In this case there is no underlying confusion, but still the plan is not to my liking. It runs counter to a generally sound strategy which I call the maxim of minimum mutilation. The classical logic of truth functions and quantification is free of paradox, and incidentally it is a paragon of clarity, elegance, and efficiency. The paradoxes emerge only with set theory and semantics. Let us then try to resolve them within set theory and semantics, and not lay fairer fields waste.

The next challenge to the law of excluded middle came out of physics: Heisenberg's paradoxical principle of indeterminacy in quantum mechanics. Certain magnitudes are incapable of being jointly ascertained, and this impossibility is a matter not merely of human frailty but of physical law. Under these circumstances it is wasteful and misleading to retain a logical apparatus that accommodates those empty questions. Birkhoff and von Neumann accordingly proposed, in 1936, a weakened substitute for truth-function logic. It lacks classical negation and, therewith, the law of excluded middle. It is not a many-valued logic; it is not truth-functional at all in structure. Alternative proposals by Rosser and by Destouches, to the same purpose, do use three-valued logic.

Most theoreticians of quantum mechanics have passed over these reforms. George Mackey has made some use of Birkhoff and von Neumann's logic. But Popper has lately argued that this logic cannot accomplish what it was meant for.

Whatever the technical merits of the case, I would cite again the maxim of minimum mutilation as a deterring consideration. I do place the claims of physics somewhat above those of set theory, because I see the justification of mathematics only in what it contributes to our integral science of nature. It is a question of remoteness from the data of observation; physics is less remote than set theory. But in any event let us not underestimate the price of a deviant logic. There is a serious loss of simplicity, especially when the new logic is not even a many-valued truth-functional logic. And there is a loss, still more serious, on the score of familiarity. Consider again the case, a page or so back, of begging the question in an attempt to defend classical negation. This only begins to illustrate the handicap of having to think within a deviant logic. The price is perhaps not quite prohibitive, but the returns had better be good.

We noticed a page back, as prompting a next to silliest objection to the law of excluded middle, a confusion between truth and knowledge. Now the present objection from quantum mechanics is in a way reminiscent of this, though without the confusion. It is an objection to any exorbitant excess of admissible questions over possible answers. Other things being equal, such an objection is sound; but we must weigh this excess against the gain in simplicity that it brings. Certainly the scientist admits as significant many sentences that are not linked in a distinctive way to any possible observations. He admits them so as to round out the theory and make it simpler, just as the arithmetician admits the irrational numbers so as to round out arithmetic and simplify computation; just, also, as the grammarian admits such sentences as Carnap's 'This stone is thinking about Vienna' and Russell's 'Quadruplicity drinks procrastination' so as to round out and simplify the grammar. Other things being equal, the less such fat the better; but when one begins to consider complicating logic to cut fat from quantum physics, I can believe that other things are far from equal. The fat must have been admirably serving its purpose of rounding out a smooth theory, and it is rather to be excused than excised.